Dark matter in the framework of shell-universe

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Abstract

We show that the shell-universe model (used to explain the observed expansion rate of the universe without a dark energy component) provides a natural mechanism for local increasing of the brane tension leading to the modified Newton's law alternative to galactic dark matter.

Kay words: Brane, Dark matter, MOND.

1 Introduction

Numerous astrophysical observations have shown that classical Newtonian dynamics fails on galactic scale and beyond, if only visible matter is taken into account. So long the only evidence for the dark matter is its global gravitational effect. The two most popular theoretical concepts dealing with this problem are dark matter (DM) and modified Newtonian dynamics (MOND). In other words, either the universe contains large quantities of unseen matter, or gravity is not generally the same as it appears to be in the solar system.

MOND, alternative to cosmic dark matter was proposed in [1] to explain flat rotation curves or the existence of a mass-rotation velocity relation for spiral galaxies. An obvious first choice would be to propose that gravitational attraction force becomes more like $1/r$ beyond some length scale, which is comparable to the scale of galaxies [2]. In this model a test particle at a distance r from a large mass M is subject to the acceleration

$$|\vec{a}| = \frac{G_N M}{r^2} g(r/r_0) \ , \tag{1}$$

where G_N is the Newtonian constant, r_0 is of the order of the sizes of galaxies (a few kpc) and $g(r/r_0)$ is a function with the asymptotic behavior

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[2]:

$$g(r/r_0) = 1 , \qquad (r \ll r_0) ,$$
$$g(r/r_0) = r/r_0 , \qquad (r \gg r_0) . \tag{2}$$

The fact that, to some extant, both DM and MOND can successfully explain galactic dynamics favors the possibility that there exists a deeper connection between these two theories [3]. Let us consider one such possibilities in the braneworld approach.

Throughout this paper we assume that galaxies containing only the observable baryonic matter are a fair sample of the universe. We want to consider the model of the universe as a 3d spherical brane expanding in a 5d space-time. This brane provides the localization of matter on the S^3 and in a large-scale approximation can be regarded as a homogenous and isotropic closed universe. The shell-universe model allows one to explain the observed expansion rate of the universe without a dark energy component [4]. Two observed facts of cosmology, the isotropic runaway of galaxies and the existence of a preferred frame in Universe where the relic background radiation is isotropic, also find their obvious explanation in shell-universe model.

Here we consider the shell-universe model on the scale characterized by the galaxy size and study the possibility how the galaxy in this model can modify Newton's law at large distances. The discussion is present in section 2. Section 3 is devoted to the summary and in the appendix we recall some results from the theory of elasticity.

2 Galaxy in/on the shell-universe

We shall consider the shell-universe model with the negative cosmological constant, which is the same inside and outside of the shell [4]. Assuming that the present value of the shell radius much exceeds the galaxy size one can approximate the shell locally (at the distances comparable to the galaxy size) by the well known flat brane solution [5, 6]. The flat brane solution embedded in AdS_5 and located at $y = y_0$ has the form

$$ds^2 = e^{-2\varepsilon|y-y_0|}\eta_{\mu\nu}dx^\mu dx^\nu + dy^2 , \tag{3}$$

where μ, $\nu = 0$, 1, 2, 3, parameter y_0 denotes the present value of the shell-universe radius, $\varepsilon = \sqrt{-\Lambda/6}$ characterizes the width of the brane and $\eta_{\mu\nu} = diag(- + + +)$ is the four dimensional Minkowskian metric. The negative bulk cosmological constant Λ is adjusted as $\kappa^4\lambda^2/6 = -\Lambda$, where λ is the brane tension and $G = \kappa^2/8\pi$ is the 5d gravitational constant

Now let us put the galaxy on the brane. In the presence of matter on the brane (3), the linearized approximation to the gravitational field requires choosing a gauge in which the brane is bent in some neighborhood of the matter [7, 8]. However, as it was shown a bit later this kind of brane

bending is the artefact of incorrect gauge fixing condition [9] and should be ruled out. Nevertheless, it is of interest to view what kind of modification of Newton's law arises due to this bending, which we parameterize as $y = y_0 - \xi(x^\mu)$.

The real bending of the brane $\delta\xi$ appears because of the gravitational attraction the galaxy by the shell-universe. The galaxy deforms some neighborhood of that part of the brane upon which it acts and leads to the appearance of an "elastic" force. We have introduced the "elasticity" of the brane into play in order to compensate the gravitational force between the galaxy and the shell-universe. Hence, in the region the galaxy is situated the energy density of the brane (i.e. the brane tension λ) is increased. Generally speaking, the increment of tension is proportional to the gravitational force by which the shell-universe attracts the galaxy and is inversely proportional to the galaxy volume. Since the core of the galaxy is much more dense than its outer part, it is natural to assume that the deformation of the brane is defined mainly by the galaxy core and thus with a good accuracy is spherically symmetric. By taking into account that the mass of the shell-universe is $2\pi^2\lambda y_0^3$ and in 5d space-time the gravitational force obeys the inverse-cube law one finds that the magnitude of gravitational force which acts on the unit of brane area is proportional to $\kappa^2\lambda T_{00}(\vec{x})$, where $\vec{x}$ are spatial coordinates on the brane and $T_{00}(\vec{x})$ denotes the matter density of the galaxy. Due to its "elastic" nature the increment of the brane tension, i.e. the deformation energy density, is equal to

$$\delta\lambda = \frac{D}{2}(\triangle\delta\xi)^2 \;, \tag{4}$$

where $\triangle$ denotes the 3d Laplace operator and D is the "stiffness" coefficient of the brane [10] (see Appendix). Using the equation describing the deformation of an elastic plate one finds that $\delta\xi$ is governed by

$$\triangle^2\delta\xi = \frac{\pi\kappa^2\lambda}{4D}T_{00}(\vec{x}) \;, \tag{5}$$

with the boundary conditions

$$\delta\xi(R) = 0 \;, \qquad \triangle\delta\xi(R) = 0 \;, \tag{6}$$

where R is the radius of concavity.

Far from the center of galaxy $(r \gg r_0)$ the eq.(5) reduces to

$$\triangle^2\delta\xi = \frac{\pi\kappa^2\lambda M}{4D}\delta(\vec{x}) \;, \tag{7}$$

where M denotes the mass of galaxy. From eqs.(4) and (7) one finds

$$\delta\xi = -b\frac{(R-r)\,(2R-r)}{3R} \;,$$

$$\delta\lambda = 2b^2D\left(\frac{1}{R} - \frac{1}{r}\right)^2 \;, \tag{8}$$

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where

$$b = -\frac{\kappa^2 \lambda M}{32D} = -\frac{3\epsilon M}{16D} \ . \tag{9}$$

From now on we assume that the coordinates y, $\vec{x}$ are Gaussian (normal) ones in some neighborhood of the 3-brane. The gravitational equations on the 3-brane take the form [11]

$$G_{\mu\nu} = -\Lambda_4 g_{\mu\nu} + \frac{\kappa^4 \lambda}{6} T_{\mu\nu} + \kappa^4 \Pi_{\mu\nu} - E_{\mu\nu} \ , \tag{10}$$

where $T_{\mu\nu}$ is the matter energy momentum tensor,

$$\Lambda_4 = \frac{1}{2} \left(\Lambda + \frac{\kappa^4 \lambda^2}{6} \right) \tag{11}$$

is 4d cosmological constant, $E_{\mu\nu}$ is limiting value of the electric part of the bulk Weyl tensor on the brane and

$$\Pi_{\mu\nu} = -\frac{1}{4} T_{\mu\alpha} T_{\nu}{}^{\alpha} + \frac{1}{12} T T_{\mu\nu} + \frac{1}{8} g_{\mu\nu} T_{\alpha\beta} T^{\alpha\beta} - \frac{1}{24} g_{\mu\nu} T^2 \ , \tag{12}$$

here $T = T_{\mu}{}^{\mu}$.

In the low energy limit the quantities $E_{\mu\nu}$ and $\Pi_{\mu\nu}$ are negligible and eq.(10) reduces to the 4d conventional Einstein gravity [11]

$$G_{\mu\nu} \simeq -\Lambda_4 g_{\mu\nu} + \frac{\kappa^4 \lambda}{6} T_{\mu\nu} \ . \tag{13}$$

Assuming $\delta\lambda(r) \ll \lambda$, in the Newtonian approximation [12]

$$T_{00} \gg |T_{\alpha i}| \ , \qquad g_{00} = -1 - 2\varphi \ , \qquad R_{00} = \triangle\varphi \ , \tag{14}$$

the eq.(13) takes the form

$$\triangle\varphi = 4\pi G_N \left[\delta\lambda(r) + T_{00} \right] \ , \tag{15}$$

where $G_N = \kappa^4 \lambda / 48\pi$.

From eq.(15) follows that the potential about a mass M has the form

$$\varphi = -G_N \frac{M}{r} - G_N \int d^3 x' \frac{\delta\lambda(r')}{|\vec{r} - \vec{r'}|} \ . \tag{16}$$

From eq.(8) one obtains that in the region $r_0 \ll r \ll R$ the tension increment behaves as

$$\delta\lambda \sim r^{-2} \ . \tag{17}$$

This leads to the appearance of a logarithmic potential far from the galaxy center. More precisely, for the tension increment $\delta\lambda$ given by eq.(8) the solution of eq.(15) in the region $r_0 \ll r < R$ behaves as

$$\varphi = 8\pi b^2 D G_N \left\{ \frac{r^2}{6R^2} - \frac{r}{R} + \ln(r/\beta) \right\} - G_N \frac{M}{r} \ , \tag{18}$$

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where β is an integration constant with units of length. Thus, the gravitational acceleration of test particle in the region $r_0 \ll r \ll R$ has the form

$$\vec{a} = -\vec{\nabla}\varphi = -\left(G_N\frac{M}{r^2} + \frac{8\pi b^2 D G_N}{r}\right)\frac{\vec{r}}{r} \ . \tag{19}$$

The compatibility of this attraction law with the eq.(1) leads to the following fine-tuning condition

$$8\pi b^2 D = \frac{M}{r_0} \ . \tag{20}$$

Substituting the expression (9) this condition takes the form

$$r_0 M = \frac{32D}{9\pi\epsilon^2} \ . \tag{21}$$

From this relation it follows that if the brane-width ϵ and the stiffness coefficient D are universal parameters on the brane the length scale r_0 (at which the deviation from Newton's law becomes essential) is inversely proportional to the mass of galaxy M. For the object with mass $\sim M_\odot \approx 10^{-11}M$ one finds that modification of Newton's law takes place out of horizon and is unobservable.

Now let us consider the second possibility of modification of the Newton's law related directly with the displacement of the brane caused by the gravitational interaction between shell-universe and the galaxy. In the RS gauge, in presence of matter on the brane, the brane is bent with respect to a coordinate system based on the flat brane. In presence of matter $T_{\mu\nu}$, the location of brane is given by $y = y_0 - \xi(x^\mu)$, where $\xi(x^\mu)$ is the solution of the equation [7, 8]

$$\partial_\mu\partial^\mu\xi(x^\mu) = \frac{\kappa^2}{6}T \ . \tag{22}$$

An additional slice deformation $\delta\xi$ results the appearance of a dark matter on the brane with the following equation

$$\triangle\delta\xi = \frac{\kappa^2}{6}(3p - \rho) \ , \tag{23}$$

which comes from (22), where p and ρ - dark matter pressure and density are assumed to be time independent quantities.

Using the eq.(5) one finds the following relation between the dark and baryonic matters

$$\triangle(3p - \rho) = \frac{3\pi\lambda}{2D}T_{00} \ . \tag{24}$$

Under assumption that dark matter is essentially pressureless the eq.(24) far from the galaxy takes the form

$$\triangle\rho = -\frac{3\pi\lambda M}{2D}\delta(\vec{x}) \ . \tag{25}$$

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Its solution with the boundary condition $\rho(r = R) = 0$ has the form

$$\rho = \frac{3\lambda M}{8D} \left(\frac{1}{r} - \frac{1}{R} \right) , \qquad (26)$$

and is not consistent with the logarithmic modification of the potential at large distances from the galaxy center. However, as it was already mentioned, this type of dark matter disappears in the correct gauge [9].

3 Summary

We have considered two possibilities related with the local deformation of a brane which could account for the dark matter in the galactic halo for 4d observer. Both types of dark matter considered here are undetectable directly from the standpoint of a 4d observer, but they lead to the modification of Newton's law.

The first possibility related with the local increment of the brane tension leads to the modification of Newton's law at large distances $r \gg r_0$, which is in agreement with the eq.(1). So, in our approach each massive object is surrounded by the elastic energy halo, which effectively plays the role of dark matter. Consider the brane as a vacuum configuration the appearance of elastic energy is some kind of vacuum polarization effect due to matter.

We notice that the present scenario may also be viable for the RS model containing two branes [6]. In this case the hidden brane attracts or repulses the galaxy and thereby leads to the deformation of visible brane in the vicinity of the region where the galaxy is situated.

The second possibility, which is related directly to the bending of a brane due to presence of a matter, does not lead to the desirable modification of the Newton's law at large distances (1). One needs not worry about this because, as it was mentioned above, this kind of bending is an artefact of incorrect gauge fixing condition [9], and thereby this possibility is merely ruled out.

Acknowledgments

Appendix

Following to the textbook [10], we briefly recall the results related with the small deformation of an elastic plate slightly adapted to the 4-dimensional case. Introducing the displacement vector $u^i(x^k)$ (i, k = 1, 2, 3), which characterizes the shift of the particles of an elastic body under the deformation, the deformation tensor for small deformations can be written in

the form

$$u_{ik} = \frac{1}{2} \left(\partial_i u_k + \partial_k u_i \right) \ . \tag{27}$$

The strength tensor corresponding to this deformation is given by

$$\sigma_{ik} = \frac{E}{1+\sigma} \left[u_{ik} + \frac{\sigma}{1-3\sigma} \delta_{ik} u \right], \tag{28}$$

where $u = u_m{}^m$, E is Young's modulus and the Poisson's coefficient σ takes values in the interval $-1 \leq \sigma \leq 1/3$. The deformation energy density reads

$$\mathcal{E} = \frac{\sigma^{ik} u_{ik}}{2} = \frac{E}{1+\sigma} \left[u^{ik} u_{ik} + \frac{\sigma}{1-3\sigma} u^2 \right]. \tag{29}$$

Consider an elastic 3-dimensional plate located at $y = 0$. Usually the internal stresses appearing under the small deformation are much greater than the force that acts on the unit of plate area. This leads to the condition $\sigma_{ik} n^k = 0$, where n^k is unit normal to the plate. By virtue of eq.(28) this condition gives

$$u_{1y} = u_{2y} = u_{3y} = 0, \qquad u_{yy} = \frac{\sigma(u_{11} + u_{22} + u_{33})}{2\sigma - 1} \ . \tag{30}$$

If displacement $u^y = -\zeta(x^1, \ x^2, \ x^3)$ is known then from eq.(30) one can find

$$u_1 = y \, \partial_1 \zeta \ , \qquad u_2 = y \, \partial_2 \zeta \ , \qquad u_3 = y \, \partial_3 \zeta \ , \tag{31}$$

and correspondingly

$$u_{11} = y \partial_1^2 \zeta \ , \qquad u_{22} = y \partial_2^2 \zeta \ , \qquad u_{33} = y \partial_3^2 \zeta \ . \tag{32}$$

To simplify our consideration we suppose σ is close to $1/3$. Under this assumption the second term in eq.(29) becomes dominant and by taking into account that $u_{yy} = -y \triangle \zeta$ the deformation energy density of an elastic plate, after integrating across the plate, takes the form

$$\mathcal{E} = \frac{D}{2} (\triangle \zeta)^2 \ , \tag{33}$$

where $\triangle$ denotes the 3d Laplace operator and D is the stiffness coefficient. For the variation one obtains

$$\delta \frac{1}{2} \int d^3 x (\triangle \zeta)^2 =$$

$$= \int d^3 x \delta \zeta \triangle^2 \zeta - \oint df^k \left(\delta \zeta \partial_k \triangle \zeta - \triangle \zeta \partial_k \delta \zeta \right) \ , \tag{34}$$

where df^k is the surface element directed along the exterior normal. The equation governing the deformation is obtained by equating $\delta \mathcal{E}$ to the variation of the potential energy of plate. If P, the force per unit area of

plate, is perpendicular to the plate then by taking into account eq.(34) the equation governing deformation takes the form

$$D\triangle^2\zeta = P \;,\tag{35}$$

with the conditions

$$\delta\zeta = \triangle\zeta = 0 \tag{36}$$

at the boundary surface.

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